A Novel Fuzzy Kalman Filter for Mobile Robots Localization

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Abstract

A new method to implement fuzzy Kalman filters is introduced in this paper. This has special application in fields where inaccurate models or sensors are involved, such as in mobile robotics. The innovation consists in using possibility distributions, instead of gaussian distributions. The main advantage of this approach is that uncertainty is not needed to be symmetric, while a region of possible solutions is allowed. The contribution of this work also includes a method to propagate uncertainty through both the process and the observation models. This one is based on quantifying uncertainty as trapezoidal possibility distributions. Finally, the way to reduce the EKF inconsistence when large number of iterations are carried out is shown.

Keywords: State Estimation, Kalman Filter, Fuzzy Logic, Mobile Robotics.

1 Introduction

Mobile robots localization is traditionally carried out using probabilistic techniques. The well known Extended Kalman Filter (EKF) provides an accurate solution to mobile robots localization. Apparently, the only condition is to initiate appropriately uncertainty matrices of the initial state estimation $\mathbf{P}(0|0)$, the process model $\mathbf{Q}(k+1)$ and the measurement model $\mathbf{R}(k+1)$.

Nevertheless, localization is done by combining incoming measures with an accurate map of the environment. This implies three main inherent problems to the EKF. First, initial maps of the environment are usually fuzzy. Second, measure uncertainty is not gaussian and, indeed, is not symmetric. Third, uncertainty propagation through non-linear equations produce accumulative errors, which have demonstrated to become important when the robot moves hundreds of meters.

At least the two first problems suggest the idea of representing uncertainty by using fuzzy logic. A novel Fuzzy Kalman filter (FKF) is presented in this paper.

Many authors propose to combine fuzzy logic and the Kalman estimation process. [4] and [5] present a probabilistic EKF for mobile robots, where fuzzy rules are used to adapt uncertainty matrices $\mathbf{Q}(k)$ and $\mathbf{R}(k+1)$. [7] applies fuzzy rules combined with a Kalman filter for mobile robots localization, but fuzzy logic is only used for sensor fusion outside the state estimation process. [6] goes further and uses fuzzy relations to represent both the observation model and the system model. Nevertheless, noise is white. [9] presents a fuzzy filter for a glucoregulatory system that has better performance that a conventional EKF, but still includes probability. Finally, [10] presents the unscented Kalman filter as a method to minimize the inconsistency problem of the EKF. This method does not use fuzzy logic, and proposes the use of higher order probabilistic measures.

While existent works on FKF focus on using fuzzy rules and fuzzy relations, we propose to include fuzzy logic in variables modelling [8]. This implies that gaussian probability distributions are replaced by possibility distributions. [11] introduces what a possibility distribution is and states the probability / possibility consistency principle. [2] formalizes management of possibility distributions using sup-min rules. These operators are replaced in this work to facilitate the implementation of the Kalman filter. Additionally, the expected value of a possibility distribution is defined in [1], but we redefine their concept of variance matrix.

In our work, possibility distributions are considered at several α-cuts (in fact only $\alpha = \{0,1\}$ are taken into account). This means that many operations can be done using interval arithmetic [3]. Along this article we show how the three problems stated above disappear or are reduced with this new approach, maintaining at least the same accuracy of the localization process.

2 The Probabilistic Approach

2.1 Uncertainty representation

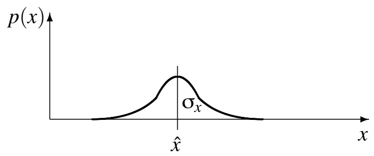
An aleatory variable is modelled by a gaussian function such as

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-\hat{x})^2}{2\sigma_x^2}}$$

with

$$\int_{-\infty}^{\infty} p(x)dx = 1$$

where $p(x)$ is the density function of x



and verifies that

$$Prob[x_a \le x \le x_b] = \int_{x_a}^{x_b} p(x)dx$$

The function is symmetric and is characterized by two parameters, the medium value,

$$\hat{x} = E[x] = \int_{-\infty}^{\infty} xp(x)dx$$

which is the most probable value, and the variance,

$$\sigma_x^2 = E[(x-\hat{x})^2] = \int_{-\infty}^{\infty} (x-\hat{x})^2 p(x)dx$$

which is a meassure of the uncertainty of x. This can be stated saying that x follows a normal distribution: $x \sim N(\hat{x}, \sigma_x^2)$. In the case of a multivariable system, a joint density function $p(x,y)$ is used, which satisfies

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x,y)dxdy = 1$$

and $Prob[(x_a \le x \le x_b) \wedge (y_a \le y \le y_b)]$ is calculated as

$$\int_{x_a}^{x_b}\int_{y_a}^{y_b} p(x,y)dxdy$$

The marginal distributions are defined as

$$p(x) = \int_{-\infty}^{\infty} p(x,y)dy$$

$$p(y) = \int_{-\infty}^{\infty} p(x,y)dx$$

The covariance between both aleatory variables is given by

$$\sigma_{x,y}^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\hat{x})(y-\hat{y})p(x,y)dxdy$$

which is 0 if x and y are independent, i.e. $p(x,y) = p(x)p(y)$, and so

$$\rho_{x,y} = \frac{\sigma_{x,y}^2}{\sigma_x\sigma_y} = 0$$

and which is $\pm\sigma_x\sigma_y$ if x and y are fully correlated,

$$\rho_{x,y} = \frac{\sigma_{x,y}^2}{\sigma_x\sigma_y} = \pm 1$$

being $\rho_{x,y}$ the correlation index. The array notation follows

$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix} \qquad \hat{\mathbf{x}} = \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}$$

$$\sigma_{\mathbf{x}}^2 = \begin{bmatrix} \sigma_x^2 & \rho_{x,y}\sigma_x\sigma_y \\ \rho_{x,y}\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix}$$

which is the variance matrix. Now $\mathbf{x} \sim N(\hat{\mathbf{x}}, \sigma_{\mathbf{x}}^2)$ and uncertainty is propagated as follows. If $z = ax + b$,

$$\hat{z} = a\hat{x} + b$$

$$\sigma_z^2 = a^2 \sigma_x^2$$

which drives to the distribution conservation principle: $dz = a\, dx$, and

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma_z} e^{-\frac{(z-\hat{z})^2}{2\sigma_z^2}} = \frac{p(x)}{|a|}$$

so

$$\int_{-\infty}^{\infty} p(z)dz = \int_{-\infty}^{\infty} p(x)dx$$

and the area is always kept equal to 1. If $z = x + y$,

$$\hat{z} = \hat{x} + \hat{y}$$

$$\sigma_z^2 = \sigma_x^2 + \sigma_y^2 + 2\rho_{x,y}\sigma_x\sigma_y$$

In general, when $\mathbf{z} = \mathbf{A}\mathbf{x} + \mathbf{b}$,

$$\hat{\mathbf{z}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{b}$$

and

$$\sigma_{\mathbf{z}}^2 = \mathbf{A}\ \sigma_{\mathbf{x}}^2\ \mathbf{A}^T$$

2.2 The Kalman filter

The Kalman filter is based on using both a process model and an observation model. In the linear case, these models are, respectively

$$\mathbf{x}(k+1) = \phi(k)\mathbf{x}(k) + \mathbf{G}(k+1)\mathbf{u}(k+1) + \mathbf{v}(k+1)$$

$$\mathbf{z}(k+1) = \mathbf{H}(k+1)\mathbf{x}(k+1) + \mathbf{w}(k+1)$$

with gaussian distributions

$$\mathbf{w}(k+1) \sim N(\mathbf{0}, \mathbf{R}(k+1)) \qquad \text{(measure noise)}$$

$$\mathbf{v}(k+1) \sim N(\mathbf{0}, \mathbf{Q}(k+1)) \qquad \text{(process noise)}$$

$$\hat{\mathbf{x}}(k|k) \sim N(\mathbf{x}(k), \mathbf{P}(k|k)) \qquad \text{(state estimation)}$$

with $\mathbf{R}(k+1) = \sigma_{\mathbf{w}(k+1)}^2$, $\mathbf{Q}(k+1) = \sigma_{\mathbf{v}(k+1)}^2$ and $\mathbf{P}(k|k) = \sigma_{\hat{\mathbf{x}}(k|k)}^2$ uncertainty matrices. $\mathbf{u}(k+1)$ is a non aleatory variable.

The Kalman filter steps follow:

a) Prediction:

$$\hat{\mathbf{x}}(k+1|k) = \phi(k)\hat{\mathbf{x}}(k|k) + \mathbf{G}(k+1)\mathbf{u}(k+1)$$

$$\mathbf{P}(k+1|k) = \phi(k)\mathbf{P}(k|k)\phi^T(k) + \mathbf{Q}(k+1)$$

$$\hat{\mathbf{z}}(k+1) = \mathbf{H}(k+1)\hat{\mathbf{x}}(k+1|k)$$

$$\begin{aligned} \mathbf{S}(k+1) &= \mathbf{H}(k+1)\mathbf{P}(k+1|k)\mathbf{H}^T(k+1) \\ &+ \mathbf{R}(k+1) \end{aligned}$$

b) Observation:

$$\mathbf{z}(k+1)$$

c) Matching:

$$\nu(k+1) = \mathbf{z}(k+1) - \hat{\mathbf{z}}(k+1)$$

with $\nu(k+1)$ the innovation array and $\mathbf{S}(k+1) = \sigma_{\nu(k+1)}^2$ its uncertainty. In the case that $\nu(k+1)$ is too big, the observation $\mathbf{z}(k+1)$ must not be used to correct the state estimation $\hat{\mathbf{x}}(k+1|k)$. A Mahalanobis distance criteriun is used as follows:

$$\mathcal{D}_M^2 = \nu^T(k+1)\ \mathbf{S}^{-1}(k+1)\ \nu(k+1) < \chi_{n_z,\alpha}^2$$

being n_z the dimension of $\mathbf{z}(k+1)$ and α a confidence value.

d) Correction:

$$\mathbf{W}(k+1) = \mathbf{P}(k+1|k)\mathbf{H}^T(k+1)\mathbf{S}^{-1}(k+1)$$

$$\hat{\mathbf{x}}(k+1|k+1) = \hat{\mathbf{x}}(k+1|k) + \mathbf{W}(k+1)\nu(k+1)$$

$$\begin{aligned} \mathbf{P}(k+1|k+1) &= \mathbf{P}(k+1|k) \\ &- \mathbf{W}(k+1)\mathbf{S}(k+1)\mathbf{W}^T(k+1) \end{aligned}$$

being $\mathbf{W}(k+1)$ the Kalman gain.

The non-linear case is commented in section 4.

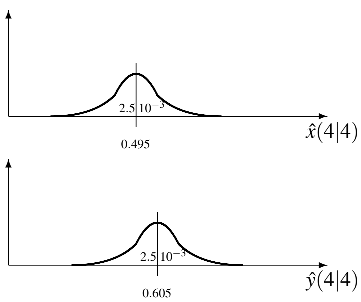
2.3 Example

Let there be the estimation of a mobile robot location

$$\hat{\mathbf{x}}(4|4) = \begin{bmatrix} 0.495 \\ 0.605 \end{bmatrix}$$

with uncertainty

$$\mathbf{P}(4|4) = \begin{bmatrix} 2.5 & 0 \\ 0 & 2.5 \end{bmatrix} 10^{-3}$$



If we suppose that the robot is completely stopped, i.e., $\mathbf{u}(5) = 0$, $\mathbf{Q}(5) = \mathbf{0}$ and $\phi(4) = \mathbf{I}$, the sensor model is given by

$$\mathbf{H}(5) = [0 \;\; 1]$$

$$\mathbf{R}(5) = 5 \; 10^{-3}$$

and the observation at $k = 5$ is 0.498, then the estimation at this instant is carried out as follows:

a) Prediction:

$$\hat{\mathbf{x}}(5|4) = \hat{\mathbf{x}}(4|4) = \begin{bmatrix} 0.495 \\ 0.605 \end{bmatrix}$$

$$\mathbf{P}(5|4) = \mathbf{P}(4|4) = \begin{bmatrix} 2.5 & 0 \\ 0 & 2.5 \end{bmatrix} 10^{-3}$$

$$\hat{z}(5) = \mathbf{H}(5) \; \hat{\mathbf{x}}(5|4) = 0.605$$

$$\mathbf{S}(5) = \mathbf{H}(5)\mathbf{P}(5|4)\mathbf{H}^T(5) + \mathbf{R}(5) = 7.5 \; 10^{-3}$$

b) Observation:

$$z(5) = 0.498$$

c) Matching:

$$\nu(5) = z(5) - \hat{z}(5) = -0.107$$

$$\nu^T(5) \; \mathbf{S}^{-1}(5) \; \nu(5) = 1.53 < \chi^2_{1,0.95} = 3.84$$

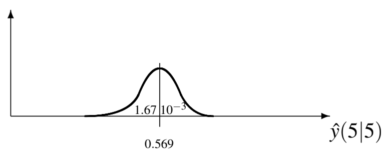
d) Correction:

$$\mathbf{W}(5) = \mathbf{P}(5|4)\mathbf{H}^T(5)\mathbf{S}^{-1}(5) = \begin{bmatrix} 0 \\ 0.333 \end{bmatrix}$$

$$\hat{\mathbf{x}}(5|5) = \hat{\mathbf{x}}(5|4) + \mathbf{W}(5)\nu(5) = \begin{bmatrix} 0.495 \\ 0.569 \end{bmatrix}$$

$$\begin{aligned} \mathbf{P}(5|5) &= \mathbf{P}(5|4) - \mathbf{W}(5)\mathbf{S}(5)\mathbf{W}^T(5) \\ &= \begin{bmatrix} 2.5 & 0 \\ 0 & 1.67 \end{bmatrix} 10^{-3} \end{aligned}$$

So the uncertainty of $\hat{y}$ decreases:

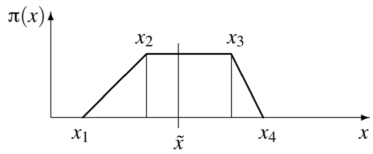


3 The Possibilistic Approach

3.1 Uncertainty representation

In many applications, variables behaviour is fuzzy rather than aleatory. For example, in mobile robotics, information obtained from the environment uses to be fuzzy. Furthermore, a person is able to navigate using qualitative criteria instead of highly precise information. Humans are able to localise themselves in the presence of uncertainty, and so a robot should be.

In order to use a possibilistic estimation algorithm, an innovative representation of the uncertainty is introduced in this section. A fuzzy variable defined over the universe of discourse X is modelled by a possibility distribution such as



$$\pi(x) = \begin{cases} 1, & \forall x \in [x_2, x_3] \\ 0, & \forall x \notin [x_1, x_4] \end{cases}$$

where now $[x_2, x_3]$ is the possible region and $[x_1, x_4]$ is the not impossible region. We represent the possibility distribution as $E[x] \sim \Pi(x_1, x_2, x_3, x_4)$. The function may not be, in general, symmetric, which is the usual case of a sensor uncertainty, for example. If we define the area of the distribution as

$$\alpha_x = \int_X \pi(x) dx$$

with $\alpha_x \neq 1$ in general, and the center of gravity as

$$\tilde{x} = \frac{\int_X x \; \pi(x) dx}{\alpha_x}$$

which can be considered an average between the most possible values and the less impossible value, then the variance,

$$\sigma_x^2 = \frac{\int_X (x - \tilde{x})^2 \pi(x) dx}{\alpha_x}$$

is a meassure of the uncertainty of x.

In the case of a multivariable system, a joint possibility distribution $\pi(x,y)$ is used, which satisfies, in general, that its volume is

$$\alpha_{x,y} = \int_X \int_Y \pi(x,y)dxdy$$

with $\alpha_{x,y} \neq 1$ in general. The marginal distributions verify that

$$\frac{\pi(x)}{\alpha_x} = \frac{\int_Y \pi(x,y)dy}{\alpha_{x,y}}$$

$$\frac{\pi(y)}{\alpha_y} = \frac{\int_X \pi(x,y)dx}{\alpha_{x,y}}$$

The dependency between both fuzzy variables is given by the covariance

$$\sigma^2_{x,y} = \frac{\int_X \int_Y (x-\tilde{x})(y-\tilde{y})\pi(x,y)dxdy}{\alpha_{x,y}}$$

which again is 0 if x and y are independent, i.e. $\pi(x,y) = \pi(x)\pi(y)$, and so $\alpha_{x,y} = \alpha_x\alpha_y$ and

$$\rho_{x,y} = \frac{\sigma^2_{x,y}}{\sigma_x\sigma_y} = 0$$

and $\pm\sigma_x\sigma_y$ (i.e. $\alpha_{x,y} = 0$) if x and y are fully correlated,

$$\rho_{x,y} = \frac{\sigma^2_{x,y}}{\sigma_x\sigma_y} = \pm 1$$

The array notation follows

$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

with $E[\mathbf{x}] \sim \Pi(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}_4)$ and variance matrix

$$\sigma^2_{\mathbf{x}} = \begin{bmatrix} \sigma^2_x & \rho_{x,y}\sigma_x\sigma_y \\ \rho_{x,y}\sigma_x\sigma_y & \sigma^2_y \end{bmatrix}$$

Uncertainty is propagated as follows. Suppose the affine relation given by $z = ax+b$. Then,

$$\pi(z) = \pi(x), \;\; \forall x,y,z \mid z = ax+b$$

This implies that

$$\forall l \in \{1,\ldots,4\}, \quad z_l = \begin{cases} a\,x_l + b & if\, a > 0 \\ a\,x_{5-l} + b & if\, a < 0 \end{cases}$$

$$\sigma^2_z = a^2\,\sigma^2_x$$

which drives again to a distribution conservation principle. Note that this problem (the sign of a) does not appear in the probabilistic case, because gaussian functions are symmetric. Anyway, the shape is maintained, instead of the area. As we can see, the main difference with respect to the probabilistic case is the distribution conservation principle.

Table 1: Distribution functions.

	Probability	**Possibility**
Area	always 1	changes
Shape	symmetric	asymmetric

On the other hand, if $z = x+y$,

$$z_l = x_l + y_l$$

and

$$\sigma^2_z = \sigma^2_x + \sigma^2_y + 2\rho_{x,y}\sigma_x\sigma_y$$

although the trapezoidal shape is not longer maintained. And finally, when $\mathbf{z} = \mathbf{A}\mathbf{x}+\mathbf{b}$,

$$\mathbf{z}_l = \mathbf{A}\mathbf{x}_l + \mathbf{b}$$

and

$$\sigma^2_{\mathbf{z}} = \mathbf{A}\;\sigma^2_{\mathbf{x}}\;\mathbf{A}^T$$

3.2 The Kalman filter

Let there be now the process and observation models

$$\mathbf{x}(k+1) = \phi(k)\mathbf{x}(k) + \mathbf{G}(k+1)\mathbf{u}(k+1) + \mathbf{v}(k+1)$$

$$\mathbf{z}(k+1) = \mathbf{H}(k+1)\mathbf{x}(k+1) + \mathbf{w}(k+1)$$

with possibility distributions

$$E[\mathbf{w}(k+1)] \sim \Pi(\mathbf{w}_1,\mathbf{w}_2,\mathbf{w}_3,\mathbf{w}_4) \quad \text{(measure noise)}$$

$$E[\mathbf{v}(k+1)] \sim \Pi(\mathbf{v}_1,\mathbf{v}_2,\mathbf{v}_3,\mathbf{v}_4) \quad \text{(process noise)}$$

$$E[\tilde{\mathbf{x}}(k|k)] \sim \Pi(\tilde{\mathbf{x}}_1,\tilde{\mathbf{x}}_2,\tilde{\mathbf{x}}_3,\tilde{\mathbf{x}}_4) \quad \text{(state estimation)}$$

with $\tilde{\mathbf{w}}(k+1) = \mathbf{0}$, $\tilde{\mathbf{v}}(k+1) = \mathbf{0}$ center of gravities, $\mathbf{x}(k) \in [\tilde{\mathbf{x}}_2, \tilde{\mathbf{x}}_3]$ and $\mathbf{R}(k+1) = \sigma^2_{\mathbf{w}(k+1)}$, $\mathbf{Q}(k+1) = \sigma^2_{\mathbf{v}(k+1)}$, $\mathbf{P}(k|k) = \sigma^2_{\tilde{\mathbf{x}}(k|k)}$ uncertainty matrices. $\mathbf{u}(k+1)$ is a non fuzzy variable.

The Kalman filter steps follow:

a) Prediction:

$$\begin{aligned} \tilde{\mathbf{x}}_l(k+1|k) &= \phi(k)\tilde{\mathbf{x}}_l(k|k) \\ &+ \mathbf{G}(k+1)\mathbf{u}(k+1) + \mathbf{v}_l(k+1) \end{aligned}$$

$$\mathbf{P}(k+1|k) = \phi(k)\mathbf{P}(k|k)\phi^T(k) + \mathbf{Q}(k+1)$$

$$\tilde{\mathbf{z}}_l(k+1) = \mathbf{H}(k+1)\tilde{\mathbf{x}}_l(k+1|k) + \mathbf{w}_l(k+1)$$

$$\begin{aligned} \mathbf{S}(k+1) &= \mathbf{H}(k+1)\mathbf{P}(k+1|k)\mathbf{H}^T(k+1) \\ &+ \mathbf{R}(k+1) \end{aligned}$$

b) Observation:

$$\mathbf{z}(k+1)$$

with

$$\mathbf{z}_l(k+1) = \mathbf{z}(k+1), \quad \forall i \in \{1, \ldots, 4\}$$

c) Matching: A possibilistic criteriun to accept or reject the observation is

$$\pi(\mathbf{z}(k+1)) \geq \alpha$$

with α a confidence value.

d) Correction:

$$\mathbf{W}(k+1) = \mathbf{P}(k+1|k)\mathbf{H}^T(k+1)\mathbf{S}^{-1}(k+1)$$

$$\begin{aligned} &\tilde{\mathbf{x}}_l(k+1|k+1) = \\ &= [I - \mathbf{W}(k+1)\mathbf{H}(k+1)]\tilde{\mathbf{x}}_l(k+1|k) + \\ &+ \mathbf{W}(k+1)\mathbf{z}_l(k+1) \end{aligned}$$

$$\begin{aligned} \mathbf{P}(k+1|k+1) &= \mathbf{P}(k+1|k) \\ &- \mathbf{W}(k+1)\mathbf{S}(k+1)\mathbf{W}^T(k+1) \end{aligned}$$

being $\mathbf{W}(k+1)$ the fuzzy Kalman gain.

3.3 Example

Let there be the same mobile robot of the previous example, located at

$$E[\tilde{x}(4|4)] \sim \Pi(0.173, 0.373, 0.673, 0.773)$$

$$E[\tilde{y}(4|4)] \sim \Pi(0.283, 0.483, 0.783, 0.883)$$

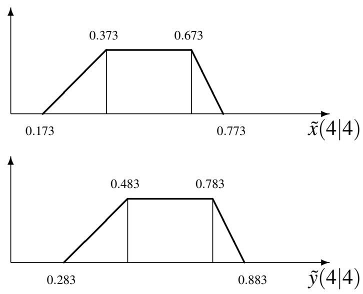
$$\mathbf{P}(4|4) = \begin{bmatrix} 0.019 & 0 \\ 0 & 0.019 \end{bmatrix}$$

If we suppose again that the robot is completely stopped, i.e., $\mathbf{u}(5) = 0$, $\mathbf{Q}(5) = \mathbf{0}$ and $\phi(4) = \mathbf{I}$,

$$E[\mathbf{v}(k)] \sim \Pi(0, 0, 0, 0)$$

and that the sensor model is given by

$$\mathbf{H}(5) = [0 \ \ 1]$$



$$E[w(5)] \sim \Pi(-0.233, -0.033, -0.033, 0.267)$$

$$\mathbf{R}(5) = 0.011$$

and that the observation at $k = 5$ is 0.498, then the estimation at this instant follows:

a) Prediction:

$$E[\tilde{x}(5|4)] \sim \Pi(0.173, 0.373, 0.673, 0.773)$$

$$E[\tilde{y}(5|4)] \sim \Pi(0.283, 0.483, 0.783, 0.883)$$

$$\mathbf{P}(5|4) = \begin{bmatrix} 0.019 & 0 \\ 0 & 0.019 \end{bmatrix}$$

$$\tilde{z}_l(5) = H(5)\tilde{x}_l(5|4) + w_l(5)$$

$$E[\tilde{z}(5)] \sim \Pi(0.05, 0.45, 0.75, 1.15)$$

$$\mathbf{S}(5) = \mathbf{H}(5)\mathbf{P}(5|4)\mathbf{H}^T(5) + \mathbf{R}(5) = 0.030$$

b) Observation:

$$z(5) = 0.498$$

$$E[z(5)] \sim \Pi(0.498, 0.498, 0.498, 0.498)$$

c) Matching:

$$\pi(0.498) = 1$$

d) Correction:

$$\mathbf{W}(5) = \mathbf{P}(5|4)\mathbf{H}^T(5)\mathbf{S}^{-1}(5) = \begin{bmatrix} 0 \\ 0.633 \end{bmatrix}$$

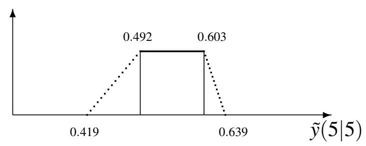
$$\tilde{\mathbf{x}}_l(5|5) = [I - \mathbf{W}(5)\mathbf{H}(5)]\tilde{\mathbf{x}}_l(5|4) + \mathbf{W}(5)\mathbf{z}_l(5)$$

$$E[\tilde{x}(5|5)] \sim \Pi(0.173, 0.373, 0.673, 0.773)$$

$$E[\tilde{y}(5|5)] \sim \Pi(0.419, 0.492, 0.603, 0.639)$$

$$\begin{aligned} \mathbf{P}(5|5) &= \mathbf{P}(5|4) - \mathbf{W}(5)\mathbf{S}(5)\mathbf{W}^T(5) \\ &= \begin{bmatrix} 0.019 & 0 \\ 0 & 0.007 \end{bmatrix} \end{aligned}$$

It can be seen that the uncertainty of $\tilde{y}$ also decreases, although the trapezoidal shape is not maintained. The example shows that, although the accuracy is not necessary to navigate, it can be also achieved using fuzzy logic.



4 Non-Linear Estimation

The probabilistic EKF covers the case in which both the process and the observation models are not linear:

$$\mathbf{x}(k+1) = \mathbf{f}(k+1, \mathbf{x}(k), \mathbf{u}(k+1)) + \mathbf{v}(k+1)$$

$$\mathbf{z}(k+1) = \mathbf{h}(k+1, \mathbf{x}(k+1)) + \mathbf{w}(k+1)$$

In this case, the equations for the estimations use the exact models,

$$\hat{\mathbf{x}}(k+1|k) = \mathbf{f}(k+1, \hat{\mathbf{x}}(k|k), \mathbf{u}(k+1))$$

$$\hat{\mathbf{z}}(k+1) = \mathbf{h}(k+1, \hat{\mathbf{x}}(k+1|k))$$

$$\nu(k+1) = \mathbf{z}(k+1) - \hat{\mathbf{z}}(k+1)$$

$$\hat{\mathbf{x}}(k+1|k+1) = \hat{\mathbf{x}}(k+1|k) + \mathbf{W}(k+1)\nu(k+1)$$

while the equations for the uncertainty matrices are approximated linearizing the models:

$$\mathbf{P}(k+1|k) = \phi(k)\mathbf{P}(k|k)\phi^T(k) + \mathbf{Q}(k+1)$$

$$\begin{aligned}\mathbf{S}(k+1) &= \mathbf{H}(k+1)\mathbf{P}(k+1|k)\mathbf{H}^T(k+1) \\ &+ \mathbf{R}(k+1)\end{aligned}$$

$$\begin{aligned}\mathbf{P}(k+1|k+1) &= \mathbf{P}(k+1|k) \\ &- \mathbf{W}(k+1)\mathbf{S}(k+1)\mathbf{W}^T(k+1)\end{aligned}$$

with

$$\phi(k) = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}(k|k), u(k+1)}$$

$$\mathbf{H}(k+1) = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\hat{\mathbf{x}}(k+1|k)}$$

This estimation has two error sources, originated by the previous linearization. The first one affects the state expectation, which is an approximation to force to obtain a symmetric (gaussian) projection. The second one affects the uncertainty matrices, which are calculated through the jacobian matrices of the non-linear functions, and which directly affects the Kalman gain calculation,

$$\mathbf{W}(k+1) = \mathbf{P}(k+1|k)\mathbf{H}^T(k+1)\mathbf{S}^{-1}(k+1)$$

which uses those uncertainty matrices and is used to correct the state prediction. These errors can lead to inconsistencies in the filter when $k \to \infty$, a phenomena that appears, for example, when a mobile robot navigates for hundreds of meters.

In the case of the proposed FKF, the first problem, i.e., the error due to projection of the center of gravity is minimised by projecting the four points of the trapezoidal funcion:

$$\tilde{\mathbf{x}}_l(k+1|k) = \mathbf{f}(k+1, \tilde{\mathbf{x}}_l(k|k), \mathbf{u}(k+1)) + \mathbf{v}_l(k+1)$$

$$\tilde{\mathbf{z}}_l(k+1) = \mathbf{h}(k+1, \tilde{\mathbf{x}}_l(k+1|k)) + \mathbf{w}_l(k+1)$$

$$\nu_l(k+1) = \mathbf{z}(k+1) - \tilde{\mathbf{z}}_l(k+1)$$

$$\tilde{\mathbf{x}}_l(k+1|k+1) = \tilde{\mathbf{x}}_l(k+1|k) + \mathbf{W}(k+1)\nu_l(k+1)$$

so

$$\tilde{z} \neq h(\tilde{x})$$

It must be taken into account that this phylosophy of uncertainty propagation can not be applied to the probabilistic Kalman filter, because its distributions are gaussian and so, they must always keep their symmetric shape.

The second problem, i.e. the error due to the uncertainty matrices, also appears as we are suppossing trapezoidal shape,

$$\alpha_z \neq h'(\tilde{x})\ \alpha_x$$

$$\sigma_z^2 \neq [h'(\tilde{x})]^2\ \sigma_x^2$$

which has also a small effect on the Kalman gain calculation.

5 Conclusion

A new FKF has been introduced which is a classic EKF able to manage possibilistic uncertainty. Its main characteristic is that fuzzy logic is included in the uncertainty model of the state estimation, instead of using fuzzy rules for process and observation models, as other authors do. Uncertainty propagation through those models has been explained, and proved to be reasonable even in the non linear case.

The main differences between the probabilistic and the possibilistic approaches are illustrated in table 2.

Table 2: Comparison between approaches.

Gaussian	**Fuzzy**
Observation noise is estimated by experimentation	Observation noise is estimated by approximation
An accurate model is needed, or a lot of measures will be rejected	Allows higher model uncertainty that will be corrected later
Strict: rejects smaller errors	It allows bigger errors
It only accepts probable data	It only rejects impossible data
It must start with an accurate estimation	By default it works with uncertain estimations

This means that the FKF has more sense in applications where uncertainty is managed in a qualitative manner, as in navigation issues in mobile robotics. Furthermore, the selection of one method does not excludes the other. In fact, the higher advantage of the presented ideas is that both types of models may be applied in parallel. While the number of iterations increase, $\hat{\mathbf{x}}(k|k)$ and $\tilde{\mathbf{x}}(k|k)$ are adjusted independently.

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